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CPTS 453

Graph Theory

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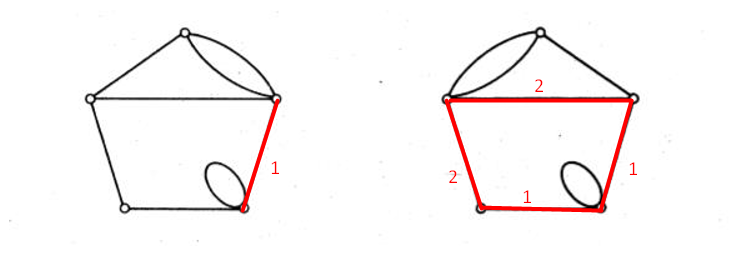
Homework 1

1.

The equation to calculate the maximum number of edges in a simple graph is. So we have which gives us = . **Therefore the maximum number of edges in a simple graph with 20 vertices is 190.**

Assuming the graph is connected then the equation to calculate the minimum number of edges in a simple graph is . Since we have 20 vertices n = 20, which gives us . **Therefore the minimum number of edges in a simple graph with 20 vertices is 0 and if it is connected then the minimum number of edges is 19.**

2.

The two graphs are not isomorphic. Both graphs have two vertices with a degree of 4, but in graph G the shortest walk from one vertex of degree 4 to the other vertex of degree 4 has a length of 1. In graph H the shortest walk from one vertex of degree 4 to the other vertex of degree 4 has a length of 2. Therefore since the lengths of the two walks between the two degree 4 vertices are different in each graph, these two graphs cannot be isomorphic.

3.

A)

Incidence Matrix M

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 14 | 15 | 16 | 24 | 25 | 26 | 34 | 35 | 36 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 6 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |

B)

Adjacency Matrix A

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 0 | 0 | 0 |
| 6 | 1 | 1 | 1 | 0 | 0 | 0 |

C)

Source for doing matrix multiplication

<https://www.dcode.fr/matrix-power>

Adjacency Matrix A6

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 243 | **243** | 243 | 0 | 0 | 0 |
| 2 | **243** | 243 | 243 | 0 | 0 | 0 |
| 3 | 243 | 243 | 243 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 243 | 243 | 243 |
| 5 | 0 | 0 | 0 | 243 | 243 | 243 |
| 6 | 0 | 0 | 0 | 243 | 243 | 243 |

**The number of walks of length 6 between vertices 1 and 2 is 243.**

D)

We calculate the Adjacency Matrix A10000

The number of walks of length 10,000 between vertex 1 and vertex 4 is:

5437833951142086247677522430603849056040441511941793313116067753972752436047730262493385210282616103224417348407745265144468527666392401284127159715122704167151064118388741463675141209614518895519886548337338059149425176663457758141815475250411845780622462535937873312408538957620672168268531058019969840153889500221039642303857257817418616875656175417293046622361693490846546977910081136717395638109639118192284819091398520739337779248542834156891019808999704734180257725329399947495758727895099796309275098928813417666481595231394558441173901477398595268756707796127643923399663613766953835665369648260688338387803448298315269779229718935895557820854346054930730221780041448044660254401864788251654483854690780167535459263528371374604973000545725678152461673313020077641731490371411964326188440805209448047394947630694488742625253500425336600510234345841946385964625247064165901766823230354151456642265325152113412259244784889046357533858323278104256551185728132049195690420147982648287411914819903734824354312588051256676736777394545177272724485539340915325018437737532838143195848515765511536532398278678848956830590642541738117652501151649467878607448576258295058603166719483945796471266031776570664364033805809468924775375483335155962912182084982886200828164845312962885514248992474789461778498694388307106200644992341698593456701435993425452285783449735288692790602021847464512033891134005993650532982326888110130501948601224749449626023670137639555778607949966541162071683434684192596616170803421278270573078983103807803941195105468361768721645557045818995131428559108257258348683656955173631198717229005717951918399116066553023397651590436115875703684157044825441995454279528653184069881794029685060957284955581544579177739174960039467262531698020285239003381362378831844139514747163145693358238719468298638045483934010815569320352919211115230565581097956541248470290017283863557619782353737524482190681696155153538942027526577228668195072761444484132052576678768793140607478606000501968401306069735656554108902317541170809126746822928038178367165577994467080073280016318911229769873440537931252385575854153883977799180088931987955307196971107353524224449949751101132624747460653591779060910012594566236249593912806582566293440533728770168777623234977101025613148263656113102100424457004695749353648950597899324371477568919762848692540943667508933518283232289236760315315534556531265784880712207410397565009107076503970984313440293254374383920185319832320387713818562759614316529096133033424563960006258049189630883395384021778406371024194139233205633998510083312578406366595586318297511455278411947718704266576054707820406605781301594134172370534402191776842616917521547665106021661966938519326648628643708117101084270418418098599027427315552932901993025830823283214727722195316948317721575632048722765936059466156817187106842060180770280248281035823122986569242278869490605686866257243390251815491146924677123742479217016140535408252509028652838649091050342466887649387037094925942701177166496304557600261055962683725718475761701287245552689801599231966919606821746224656960886720761702533073806247885668957512614225961236269171950132563937477960187257550788282432339836008616321611294408388556328978360424865431327707288015425756196800526736353329500170420823389050165087302712229564620441360519682112096321012363437344507466907453627498309394269691802304773319014975834806932426039279225810482326361918810996917860862700813413537013028172133696649622811117916148034879911358173055022544313806449328541478522447357605303584828041270972495770979426890134409396271001326128027110747494882190472524801617654225055205367175789141884956502904000596131282057260152485321015254981188654962506132160432053300823998911836284764321512265593134792410321907787208394010266099244928618202020107806363882237046226163642946702483307818944921254371408666923438977081222233141898276060385016281053793610689281826856586152583696880332002414006085133542425636062694200598506722351567109267455663249027611827035181292815629965410232030627214515825384999524671969132019826228326191393495599639479547721539513783896419459680775495889826577794904332251417234602679276212199884815909265308198794094408907834462180197107089534407630935854169933671921724786542596119293868788706344317276622758205846648973740443428085675686217720764160895559758422254669960661863549511514979741432229930398451927735322830807545425885578446162766368079353429736463551489105430530406570955350887115675618420429793745627021022739749688255695498394725605543025701222377168354988063636816732167392990825787293766869562864594169931260485723465632498969620631011544965238372371533838218127115713030781944490742164552310332015005451936054328321734553839727400715138288853220739598988109739080499368402184066667

E)

This is the Adjacency Matrix A5

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | **0** | 0 | 0 | 81 | 81 | 81 |
| 2 | 0 | **0** | 0 | 81 | 81 | 81 |
| 3 | 0 | 0 | **0** | 81 | 81 | 81 |
| 4 | 81 | 81 | 81 | **0** | 0 | 0 |
| 5 | 81 | 81 | 81 | 0 | **0** | 0 |
| 6 | 81 | 81 | 81 | 0 | 0 | **0** |

This matrix shows all number of possible walks from one vertex to another. Since A1,1 A2,2 A3,3 A4,4 A5,5 and A6,6 all have 0 then **it must be the case that there does not exist a closed walk of length 5 for this graph.**

F)

**They are not isomorphic,** since K3,3 is bipartite and the graph shown is not bipartite. One graph can’t be bipartite and the other not be bipartite and be isomorphic at the same time.